

Comparative Analysis of Direction of Arrival Estimation algorithms

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Abstract: Direction of Arrival (DOA) estimation has typically played a key role in signal processing. Its task is to find the directions impinging on an antenna array to increase the performance of the received signal. It has become the key of technique implementation to use DOA estimation methods that are applicable to most environments. This estimation is an efficient method for improving the quality of service in a communication system by focusing the reception and transmission only in the estimated direction. The traditional algorithms such as MUSIC, ESPRIT, etc. can get superior performance for DOA estimation in the rich receiving conditions, but they are not fit for adverse environment such as low SNR, small number of array elements or snapshots, etc. In order to improve the performance of DOA estimation, the modified method is based on wavelet operator. Moreover, the method is applied to MUSIC (Multiple Signal Classification) DOA estimation algorithm to get the modified algorithm WMUSIC. Undoubtedly, the wavelet based method expands the application range of traditional DOA estimation algorithms and has widely practical prospects in future. Further results are improved to minimize the errors in angle estimation by using optimization techniques.

Keywords: Direction of Arrival, MUSIC, SNR.

I. INTRODUCTION

The array signal processing issue is developing very rapidly from the last decades. DOA estimation has been widely applied in many actual domains as one of most critical technology of array signal processing. Domains such as speech signal processing, radar, mobile communication and so on use this technology [1]. The main technologies focus on DOA estimation in the application of smart antennas. The accuracy of DOA estimation generally will affect the consequence of beamforming immensely [2].

The high accurate DOA estimation will bring many significant advantages like increasing the capacity, transmitter power reduction of mobile terminal, resistance to multipath effect, decreasing interference from outside, and likewise [3]. It is seen due to the importance of DOA estimation the work presented before are concerted in exploring the high resolution for DOA estimation. In recent years various algorithms are proposed successively, including high-order cumulate method, capon, propagator method, min-norm, MUSIC and ESPRIT. MUSIC is considered to be high resolution and accuracy method [4], [5]. From the previous studies it is seen that the performance of the most DOA estimation algorithms is dependent on some factors. These factors are viz: spatial distribution and the number of array elements, the number of snapshots and signal-to-noise ratio (SNR) of received array signals [6].

Generally, number of array elements or snapshots should be large in DOA estimation for the high resolution and accuracy, but due to this there is increase in equipment overhead and also the computing complexity increases.

Usually, when the communication environment is very poor and a larger scale of snapshots or array elements is not employed then even by adopting MUSIC satisfactory performance of DOA estimation can't be obtained. But to increase the number of snapshots or array elements in real time is somewhat difficult. Therefore, high performance estimation algorithms of DOA should be based on the improvement of SNR of the receiving signal only in the limited hardware conditions.

II. SIGNAL MODEL

Most of the DOA estimation methods which based on signal processing rely on certain assumptions made on the received antenna array signals. In this section, the 2D pattern is used to describe briefly the signal received model of DOA as follows.

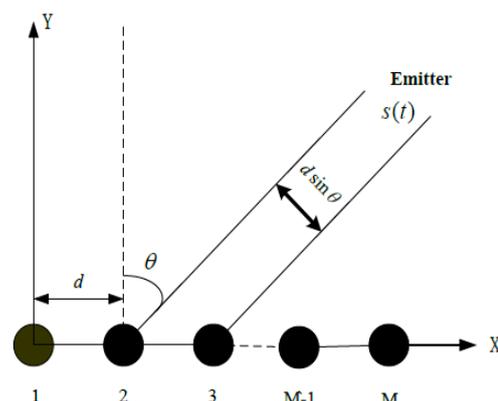


Fig.1. Antenna array of M element with θ arriving signal

We have considered a scenario with emitting sources from various directions with narrowband property. We have a linear antenna array with M elements to receive each emitting signal separated by distance d as shown in Figure 1. The array is receiving a signal impinging on array axis at an angle θ . Therefore the signal of direction whose element is expressed as and the output signals of received antenna array at the sampling time can be showed as,

$$x(k) = \sum_{i=1}^d a_i s_i(k) + n(k) = A \cdot s(k) + n(k) \quad (1)$$

Where $A = [\alpha(\theta_1), \alpha(\theta_2), \dots, \alpha(\theta_k)]$, $s(k) = [s_1(k), s_2(k), \dots, s_D(k)]^T$, and the signal wavelength. Moreover $n(k) = [n_1(k), n_2(k), \dots, n_M(k)]$ is assumed to be spatially white Gaussian noise with variance. Generally, the designed antenna arrays require that $M > D$.

III. MUSIC DOA ALGORITHM

Multiple Signal Classification (MUSIC) is the most popular technique used in Direction of arrival estimation. We can summarize DOA estimation as the estimation work of the direction of an unknown incoming signal to a receiving antenna by some processing techniques.

This method is a relatively simple and efficient Eigen structure method of direction of arrival estimation. It has many variations and is perhaps the most studied method in its class. This method is also known as spectral MUSIC, which estimates the noise subspace from available samples. The same can be done by either Eigen value decomposition of the estimated antenna array correlation matrix or singular value decomposition of the data matrix, with its N columns equals to N snapshots of the array signal vectors. When the noise subspace has been estimated, a search for an angle pairs in the range is made by looking for steering vectors orthogonal to the noise subspace as possible. Normally this is accomplished to search for peaks in the MUSIC spectrum.

Multiple Signal Classification (MUSIC) relies on the array correlation matrix. Assuming ergodicity, the time-averaged array correlation matrix of is given by,

$$\widehat{R_{xx}} = \frac{1}{K} \sum_{k=0}^{K-1} x(k)x^H(k) \quad (2)$$

where k is the total number of samples. MUSIC is a subspace method that can potentially provide high resolution by exploiting the structure of the input data model. The MUSIC spectrum is computed by

$$P_{MU}(\theta, \phi) = \frac{1}{\alpha^H(H_N)E_N^H \alpha(\phi)} \quad (3)$$

Where E_N represents the noise eigenvectors [6] of $\widehat{R_{xx}}$. In order to determine E_N , it is necessary to separate the signal subspace from the noise subspace. This separation can be achieved either by using a threshold or by more advanced techniques such as more robust performance. For the

simulations involving MUSIC the number of incoming signals for the MUSIC algorithm is assumed to be known. Multiple Signal Classification (MUSIC) method was proposed by Schmitt and his colleagues in year 1979. It has created a new era for spatial spectrum estimation algorithms. The promotion of the structure algorithm characterized rise and development, and it has become a crucial algorithm for theoretical system of spatial spectrum. Before this algorithm was presented, some relevant algorithms directly processed data received from array covariance matrices. The basic idea of MUSIC algorithm is to conduct characteristic decomposition for the covariance matrix of any antenna array output data that results in a signal subspace orthogonal with noise subspace corresponding to the signal components. Then these two orthogonal subspaces are used for constituting a spectrum function, gone though by spectral peak search and detect DOA signals. It is because MUSIC algorithm has a high resolution, accuracy and stability under certain conditions that it attracts a large number of scholars to conduct in-depth research and analysis.

In general, it has the following advantages when it is used to estimate a signal's DOA:

- The ability to simultaneously measure multiple signals.
- High precision measurement.
- High resolution for antenna beam signals.
- Applicable to short data circumstances.
- It can achieve real-time processing after using high-speed processing technology.

1. Flow chart of MUSIC Algorithm:

Sensor array matrix X is an l by n matrix where l is the number of sensor antennas and n is the number of snapshots taken. Matrix can be formulated as given in the equation 4.

$$X^T = [x_1 \dots x_l] \quad (4)$$

Total signal induced on the l^{th} element of the receiver array can be formulated by the equation 5

$$x_l = \sum_{k=1}^K m_k(t) e^{j2\pi f_0 \tau_l(\theta_k)} + n_l(t) \quad (5)$$

Time taken (used as time delay) by the signal to reach the l^{th} element of the receiver array from the reference array element by the k^{th} signal coming from (θ_k) can be calculated by the equation 6

$$\tau_l(\theta_k) = \frac{\bar{r}_l \cdot \tilde{v}(\theta_k)}{c} \quad (6)$$

where \bar{r}_l denotes the position vector of l^{th} antenna $\tilde{v}(\theta_k)$ denotes the unit vector directed to k^{th} incoming signal. The autocorrelation matrix of the sensor array can be obtained by the equation 7

$$R = E\{XX^T\} \quad (7)$$

Find the correlation matrix of the receiver antenna array elements by using the formula given in the equation 8.

$$R = \frac{1}{K} \sum_{n=1}^N x_n x_n^H \quad (8)$$

Calculate eigen values and eigen vectors of the correlation matrix using equation 8. Compose a noise subspace matrix which is eigen vectors that corresponds to smallest eigen values of the correlation matrix. For both of all the theta and phi angles, create steering vector by using the formula given in equation 9

$$S(\theta) = [e^{2\pi f_0 \tau_2(\theta)} \dots e^{2\pi f_0 \tau_1(\theta)}] \quad (9)$$

Calculate P_{MU} for all angle values by using 10 Peaks of the MUSIC spectrum are the estimated DOA angles.

$$P_{MU}(\theta) = \frac{1}{|S^H(\theta) U_L|^2} \quad (10)$$

Where U_L denotes an l by $l - m$ dimensional matrix with its $l - m$ columns being eigenvectors corresponding to the $l - m$ smallest eigen values of the array correlation matrix. $S^H(\theta)$ is the hermitian (transpose of complex conjugate) of the steering vector that is used for scanning the range of meaningful angles for the user.

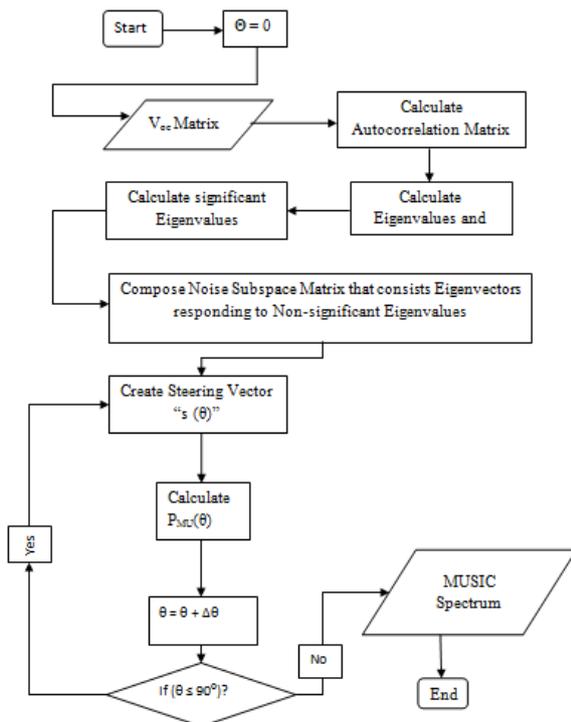


Fig. 2: Flowchart of Multiple Signal Classification Algorithm

IV. WMUSIC DOA ALGORITHM

As many research results have demonstrated [8], MUSIC algorithm can get good performance on DOA estimation when the communication environment is normal (SNR is

not very low). However, this algorithm will gradually deteriorate when the SNR continues to decrease. To solve this problem, we utilize the wavelet operator to denoise for improving the SNR of received signals in this section. And then it is applied to the MUSIC algorithm named WMUSIC algorithm. The algorithm can be specifically described as follows.

According to Mallet theory [12] the array signals $x(k)$ in (1) are decomposed to T layer by discrete wavelet. Then the characteristics coefficients $q_{j,t}$ and $p_{j,t}$ of the array signals in the T scale decomposition model can be denoted by

$$\begin{cases} q_{j,t} = \langle x(k), \zeta_{j,t}(k) \rangle \\ p_{j,t} = \langle x(k), \xi_{j,t}(k) \rangle \end{cases} t \in Z \quad (11)$$

Where $\xi(k)$ and $\zeta(k)$ are respectively scaling function and wavelet function. Furthermore p_T is the approximation coefficients of T scale, and mainly presents low-frequency property of the array signals. Correspondingly, q_j is the detail coefficients of j scale, and mainly presents high-frequency property of the array signals. t is the time shift exponent. The array signals $x(k)$ can be refresh expressed by a linear combination with p and q such that

$$x(k) = \sum_t p_{T,t} \xi_{M,t}(k) + \sum_{i=1}^T q_{j,t} \cdot \zeta_{j,t}(k) \quad (12)$$

Most of white Gaussian noise is included in the components with high-frequency property. The $\xi_{T,t}(k)$ and $\zeta_{j,t}(k)$ can be obtained by mother wavelet $\xi(k)$ and scaling function $\zeta(k)$ after extending such as,

$$\begin{cases} \xi_{T,t}(k) = 2^{-\frac{T}{2}} \cdot \xi(2^{-T}k - t) \\ \zeta_{j,t}(k) = 2^{-j/2} \cdot \zeta(2^{-j}k - t) \end{cases} \quad (13)$$

The low-frequency coefficient $p_{j,t}$ and the high-frequency coefficient $q_{j,t}$ of j scale can be gotten by firstly convoluting through low-pass filter $h(m)$ and high-pass filter $g(m)$, then executing sampling processing such that

$$\begin{aligned} q_{j,t} &= \sum_m h(m - 2t) \cdot p_{j-1,t} \\ p_{j,t} &= \sum_m g(m - 2t) \cdot p_{j-1,t} \end{aligned} \quad (14)$$

According to wavelet denoising principle, most of noise which is included in can be removed by choosing the appropriate threshold of high-filter $g(m)$ to filter high-frequency composition. Based on the empirical formal, the threshold can be used as

$$\epsilon = \sigma_{thr} * \sqrt{2 \log(k)} \quad (15)$$

Where σ_{thr} is the variance of high-frequency composition in the highest scale M and k is the length of high-frequency composition. Completed the filtering process for high-frequency composition, the high-frequency coefficient $q_{j,t}$ is renewed.

Then the low-frequency coefficient $P_{j-1,t}$ and $j-1$ of scale is reconstructed by and $q_{j,t}$ such as

$$p_{j-1,t} = \sum_t h(m - 2t)p_{j,t} + \sum_t g(m - 2t) \cdot q_{j,t} \quad (16)$$

Finally, the original array signals $x(k)$ can be replaced by denoising array signals $\bar{x}(k)$ which is single-reconstructed by the low-frequency coefficient $\bar{p}_{j-1,t}$ and the high-frequency coefficient $\bar{q}_{j,t}$. Then the processed array signals $\bar{x}(k)$ is used to MUSIC to build new algorithm named WMUSIC.

V. SIMULATION RESULTS

The first result shows the simulation for basic MUSIC algorithm. There are two coherent signals, the incident angle is 20° and 55° respectively, the array spacing is $\lambda/4$ and $\lambda/6$, the noise is ideal Gaussian white noise, the SNR is 15dB, number of array element is 20 and the number of snapshots is 200. The simulation results are shown in figure 3.

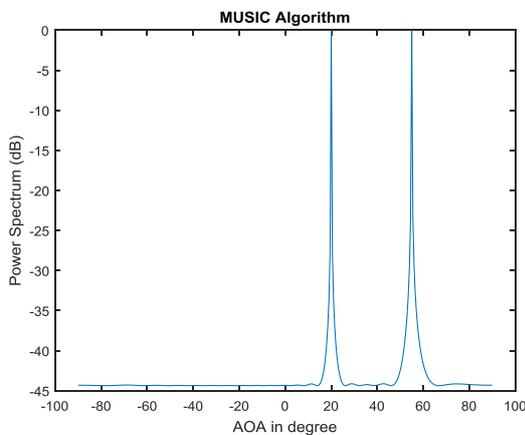


Fig. 3: Simulation for basic MUSIC algorithm

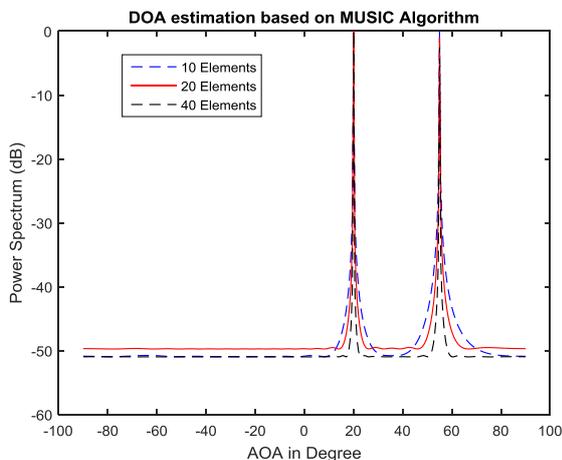


Fig. 4: Simulation for MUSIC algorithm with varying number of elements

Figure 4 shows the simulation for same specification of basic MUSIC algorithm with varying number of elements

as 10, 20, and 40. As shown in figure 4, performance of the algorithm increases as the number of array element increases.

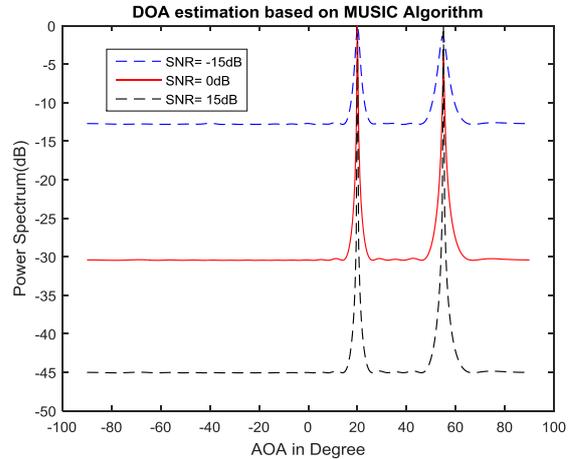


Fig. 5: Simulation for basic MUSIC varying SNR

Figure 5 shows the performance of this algorithm in an environment with varying SNR = -15dB, 0dB and 15dB. For low values of SNR, the spikes depicting the arrival of a signal from certain direction are small. It is thus difficult to exactly extract the angle of arrival. As the values of SNR increase, however, the resolution of the algorithm is observed to improve considerably and the spikes become more definite. This is attributed to the fact that for low SNR the difference between the eigenvalues associated with the signal and those associated with noise become smaller and the peaks therefore become smaller with respect to the noise levels. With increase in SNR, the difference between the two sets of eigenvalues is substantial and the peaks are bigger with respect to the noise levels.

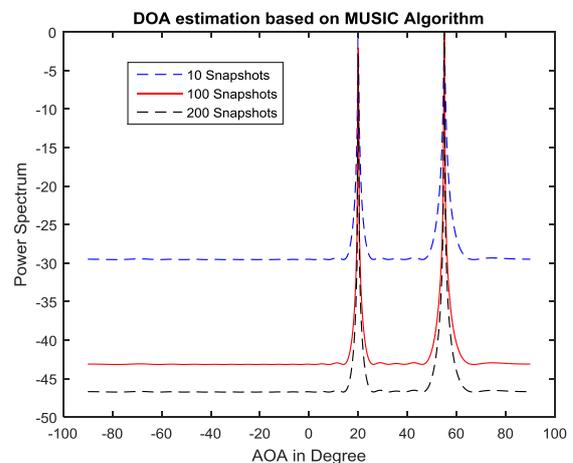


Fig. 6: Simulation for basic MUSIC varying snapshots

Figure 6 shows the simulation for same specification of basic MUSIC algorithm with varying number of snapshots as 10, 100 and 200. As shown in figure 6, performance of the algorithm increases as the number of samples increases.

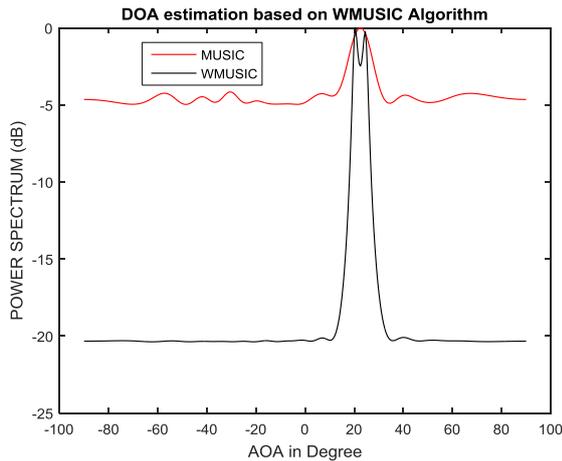


Fig. 7: Simulation for WMUSIC

Compared with MUSIC, the proposed WMUSIC algorithm has a much better performance. As shown in the figure 7, at SNR= -8dB, the near desired angles 20° and 25° can be distinguished clearly by WMUSIC algorithm.

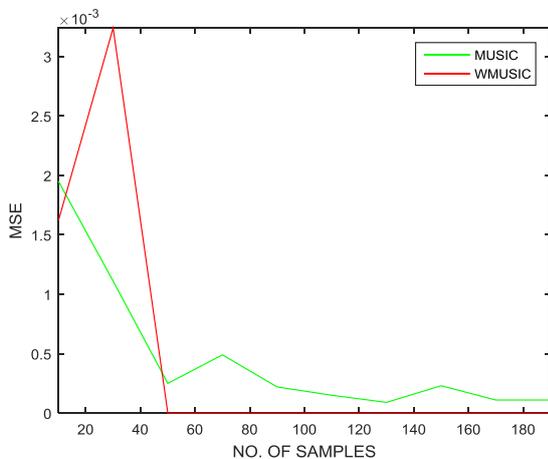


Fig. 8: Average RMSE vs No. Of samples of two algorithms

In order to fully compare the performance, we test the root mean square error (RMSE) of DOA estimation for the two algorithms, i.e. MUSIC and WMUSIC.

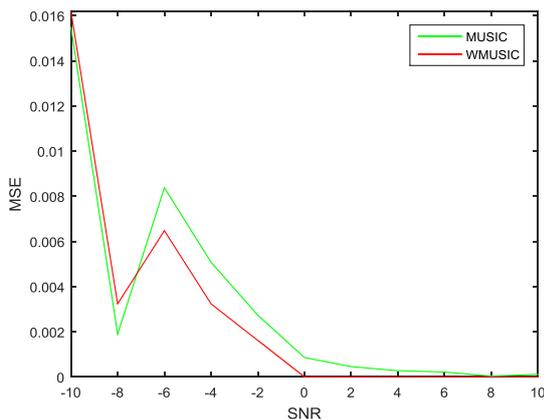


Fig. 9: Average RMSE vs SNR of two algorithms

To get the average RMSE results of DOA estimation each test under varying SNR (-10dB to 10dB) and varying snapshots (10 to 200) is repeated as shown in figure 8 and figure 9. With the increase in SNR, the average RMSE of each algorithm decline accordingly for DOA estimation. Compared against MUSIC, the proposed WMUSIC algorithm can obtain higher accuracy in lower SNR (-10,-3,0,10). However, when the SNR reach enough (SNR>6), the advantage of the new algorithms may be lost.

VI. CONCLUSION

In this paper two DOA algorithms are developed in MATLAB software and their response to SNR is presented. This work is a novel approach attempt to solve the DOA estimation problems on poor communication conditions using the new method based on wavelet operator. The simulation results show that the new algorithm can greatly improve the performance on resolution and accuracy to compare with traditional MUSIC in low SNR environment. The proposed method can be widely applied in the design of smart antenna system. The two algorithms are high resolution algorithms and this is derived from the precision with which the angles of arrival are estimated. The two algorithms are highly sensitive to the signal to noise ratio SNR where the resolution of the algorithms is found to improve with the increase in SNR. MUSIC algorithm is suitable for an array with high SNR with average snapshots whereas WMUSIC algorithm on the other hand can perform relatively well than MUSIC in an environment having low SNR.

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